数理解析研究所講究録
1014巻 1997年 93-103

Geometric methods in Asymptotic Analysis
R.I.M.S. KYOTO : 20th - 23th May 1997

CONTACT GROUPOIDS AND PREQUANTIZATION

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Introduction. The geometric prequantization of a general Poisson manifold (Γ_0,Λ_0), as defined by Weinstein [16] and Xu, is an extension of Souriau's process [14] for symplectic manifolds. First, if possible, is associated to (Γ_0,Λ_0) a symplectic groupoid $(\hat{\Gamma},\sigma)$ [2], which is a kind of nice desingularisation, and secondly the symplectic manifold is eventually prequantized [14], that is to say, one looks to the existence of a $\mathbb{S}^1$-principal bundle $\pi : \Gamma \to \hat{\Gamma}$, with a connection λ, the curvature of which is $\pi^*\sigma$. If $\hat{\Gamma}$ is Hausdorff (which is a very special case), there exists a prequantization iff $[\sigma] \in H^2(\hat{\Gamma},\mathbb{Z})$.

Actually, if (Γ_0,Λ_0) is prequantizable, (Γ,λ) has a nice structure : it is a contact groupoid, a notion which enlarges previous one due to Libermann [11] and Kerbrat and Souici [9].

This lecture is a survey of relations between contact groupoids and prequantization [4].

Acknowledgements

I thank Professor T. KAWAI and R.I.M.S.-KYOTO for their invitation to present this lecture at the workshop they organized at R.I.M.S.

1. CONTACT GROUPOIDS [4]

Definition 1.1 - *A contact groupoid* $(\Gamma,\mathcal{H})$ *is a Lie groupoid endowed with a compatible contact structure.*

• <u>Γ is a Lie groupoid</u> : Γ is a C^∞ (generally not Hausdorff) manifold, Γ_0 is a Hausdorff submanifold of Γ (the manifold of units), α and β two C^∞ surjective submersions of Γ on Γ_0 (α is the source and β the target) ; moreover on Γ is defined a C^∞ "inverse" map $i : x \to i(x) = x^{-1}$, and on $\Gamma_0 = \{(x,y) \mid \alpha(x) = \beta(y)\}$ a C^∞ product $m : (x,y) \to m(x,y) = x.y$ such that

(i) $\forall x \in \Gamma \qquad x.\alpha(x) = \beta(x).x = x$

(ii) If $(x,y) \in \Gamma_2$ and $(y,z) \in \Gamma_2$, $(x.y,z)$ and $(x,y.z)$ are in Γ_2 and $(x.y).z = x.(y.z)$

(iii) $\forall x \in \Gamma, \qquad x^{-1}.x = \alpha(x) \qquad x.x^{-1} = \beta(x).$

First examples of Lie groupoids

(i) If Γ_0 is reduced to a point, Γ is a Lie group.

(ii) If Γ_0 is any Hausdorff C^∞ manifold, $\Gamma_0\times\Gamma_0$ is canonically endowed with a groupoid structure (the coarse groupoid of Γ_0) with the laws :

$$((x,y)(y,z)) = (x,z)$$
$$(x,y)^{-1} = (y,x)$$

and Γ_0 is identified to the diagonal of $\Gamma_0\times\Gamma_0$.

(iii) Any vector bundle $\pi : E \to \Gamma_0$ is a vector groupoid with addition in the fibers. In this case $\alpha = \beta = \pi$.

(iv) If $\Gamma \underset{\beta}{\overset{\alpha}{\rightrightarrows}} \Gamma_0$ is any Lie groupoid, the tangent groupoid of Γ, $T\Gamma$ is the Lie groupoid $T\Gamma \underset{T\beta}{\overset{T\alpha}{\rightrightarrows}} T\Gamma_0$ with the inverse law $X \to \Theta X = Ti(X)$ and the product law : $Tm : (T\Gamma_2) \to T\Gamma \quad (X,Y) \to X\oplus Y = Tm(X,Y)$.

• **Compatibility condition of Γ and $\mathcal{H}$**

$(\Gamma,\mathcal{H})$ is a contact groupoid iff

(i) $X \in \mathcal{H} \Rightarrow \Theta X \in \mathcal{H}$

(ii) $(X,Y) \in (\mathcal{H}\times\mathcal{H}) \cap T\Gamma_2 \Rightarrow X\oplus Y \in \mathcal{H}$

The first result is the following.

Theorem 1.1 [4] *If the dimension of* Γ, dim Γ, *is strictly greater than the dimension of* Γ_0,n,

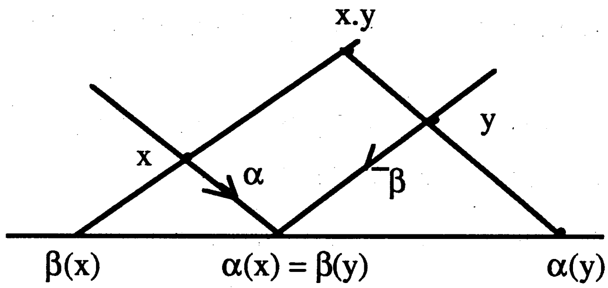
(i) dim $\Gamma = 2n+1$

(ii) Γ_0 *is a Legendrian submanifold of* $(\Gamma,\mathcal{H})$

(iii) $\mathcal{H}_\alpha = \mathcal{H} \cap$ Ker $T\alpha$ *(resp.* $\mathcal{H}_\beta = \mathcal{H} \cap$ Ker $T\beta$*) is right (resp. left) invariant.*

The notion of left (or right) invariance is explicited in the next paragraph.

2. LIE ALGEBROID OF A CONTACT GROUPOID.



If $x \in \Gamma$ the left action by x, L_x is defined only from $\beta^{-1}(\alpha(x))$ into $\beta^{-1}(\beta(x))$:

$$L_x(y) = x.y$$

This leads to the notion of left invariance for objects in Ker $T\beta$, which makes clear theorem 1.1. In particular if $X \in \mathcal{X}(\Gamma)$ Lie algebras of vector fields of Γ, X is left invariant iff $T\beta X \equiv 0$ and

$$L_x X_y = X_{xy},$$

which is equivalent to

$$X \in \mathcal{X}^{\ell}(\Gamma) \text{ iff } 0 \oplus X = X$$

that is to say $X_{xy} = 0_x \oplus X_y$.

For left invariant vector fields we have the three nice properties

(i) $\mathcal{X}^{\ell}(\Gamma)$ is a sub Lie algebra of $\mathcal{X}(\Gamma)$

(ii) Even if Γ is not Hausdorff, it can be proved that for each x, $\beta^{-1}(x)$ is a Hausdorff submanifold so if $X \in \mathcal{X}^{\ell}(\Gamma)$ it has a (local) flow.

(iii) To $X \in \mathcal{X}^{\ell}(\Gamma)$ is associated a unique vector field on Γ_0, X_0 such that $T\alpha \circ X = X_0 \circ \alpha$ and X is complete iff X_0 is.

This leads to the notion of Lie algebroid due to Pradines [13] and which plays for Lie groupoid the same role than Lie algebras for Lie groups.

Definition 2.1. *A Lie algebroid* $E \to \Gamma_0$ *is a* C^∞ *vector bundle over a* C^∞ *Hausdorff manifold together with a Lie algebra structure on the space* $\mathfrak{A}(E)$ *of sections of* E *and a vector bundle morphism, the anchor map* ρ*, from* E *to* $T\Gamma_0$ *such that, for each* $s \in \mathfrak{A}(E)$ *and* $u \in C^\infty(\Gamma_0, \mathbb{R})$

(i) $\rho_o\{s_1,s_2\} = [\rho_o s_1, \rho_o s_1]$
(ρ *is a Lie algebra morphism from* $\mathfrak{A}(E)$ to $\mathcal{X}(\Gamma_0)$)

(ii) $\{s_1, u.s_2\} = u.\{s_1,s_2\} + (\mathcal{L}_{\rho_o s_1} u).s_2$ where $\mathcal{L}$ is the Lie derivative.

Note that this implies that $\mathfrak{A}(E)$ is a class of Kirillov's algebras [10].
Going back to Γ, the Lie algebroid $\underline{\Gamma}$ of Γ can be described :

(i) As vector bundle $\underline{\Gamma}$ is the normal bundle of Γ_0 in Γ : $\underline{\Gamma} = T\Gamma|_{\Gamma_0}/T\Gamma_0$. So $\underline{\Gamma}$ is canonically isomorphic to $\mathrm{Ker}\, T\beta|_{\Gamma_0} \to \Gamma_0$.

(ii) The anchor map is defined by the diagram :

$$\begin{tikzcd} \underline{\Gamma} \arrow[rr, "\simeq"] \arrow[dr, "\rho"'] & & \mathrm{Ker}\, T\beta|_{\Gamma_0} \arrow[dl, "T\alpha"] \\ & T\Gamma_0 & \end{tikzcd}$$

(iii) The Lie algebra structure of $\mathfrak{A}(\underline{\Gamma})$ comes form the canonical isomorphisms

$$\mathcal{X}^{\ell}(\Gamma) \simeq \mathcal{X}^{\ell}(\Gamma)|_{\Gamma_0} \simeq \mathfrak{A}(\underline{\Gamma}) .$$

In the sequel we shall restrict ourselves to the special case of "oriented" contact groupoids, which is enough to our purpose. In this case $\mathcal{H}$ is the Kernel of a contact form λ, which, more over, can be choosen such that :

$$\lambda(X_x \oplus Y_y) = f(x)\lambda(Y_y) + g(y)\lambda(X_x)$$

where f and g are morphisms of Γ into R_*, multiplication group of non zero real numbers. Such a groupoid (Γ,λ) is called a *pfallian groupoid*, subordinate to $(\Gamma,\mathcal{H})$.

Theorem 2.1. [4] *If* $(\Gamma,\mathcal{H})$ *is an oriented contact groupoid and* $(\underline{\Gamma},\lambda)$ *a subordinate pfallian groupoid,*

(i) There exists on Γ_0 *a canonically defined Jacobi structure* $(\Gamma_0, E_0, \Lambda_0)$ [8].

(ii) $\underline{\Gamma}$ *is the Lie algebroid canonically defined by* (Γ_0,E_0,Λ_0) *: as vector bundle,* $\underline{\Gamma} = \mathfrak{J}^1(\Gamma_0,\mathbb{R}) \simeq \mathbb{R} \times T^*\Gamma_0$*; the anchor map* ρ *is defined by* $\rho(u,\xi) = uE_0 + \Lambda_0^{\#}\xi$ *and the bracket is the Kerbrat-Souici's bracket [9] :*

$$\{(u,\xi),(v,\eta)\} = (\{u,v\} - \iota_{\Lambda_0}(\xi-du)\wedge(\eta-dv),\ u\mathfrak{L}_{E_0}\eta - v\mathfrak{L}_{E_0}\eta - \iota_{E_0}(\xi\wedge\eta) + [\xi,\eta]_p)$$

where $\{u,v\}$ *is the Jacobi bracket on* $C^\infty(\Gamma_0,\mathbb{R})$,

$$\{u,v\} = u\mathfrak{L}_{E_0}v - v\mathfrak{L}_{E_0}u + \iota_{\Lambda_0}(du\wedge dv)$$

and where $[\xi,\eta]_p = \iota_{\Lambda_0^{\#}\xi}d\eta - \iota_{\eta_0^{\#}\xi}d\xi + d\iota_{\Lambda_0}(\xi\wedge\eta)$.

We have noted $\Lambda_0^{\#}$ the morphism $T^*\Gamma_0 \to T\Gamma_0$ given by $\iota(\Lambda_0^{\#}\xi)\eta \equiv \iota_{\Lambda_0}(\xi\wedge\eta)$.

Remark. If (Γ_0,E_0,Λ_0) is a Jacobi manifold, generally speaking there exists only a "local" Lie algebroid in Van Est's sense [4]. We say that (Γ_0,E_0,Λ_0) is *contact-integrable* if it is the units of a contact groupoid.

3. Lie groups of infinite dimension.

Before describing the relations between contact groupoids and prequantization, it is necessary to recall a notion introduced by Ehresmann [6] and which enlarges the usual notion of graph of a diffeomorphism. If $\Gamma \rightrightarrows \Gamma_0$ is a Lie groupoid, a *bisection* of Γ is a submanifold S such that $\alpha|_S$ and $\beta|_S$ are diffeomorphisms on Γ_0. So a bisection of Γ is, equivalently, a section S^β of $\beta : \Gamma \to \Gamma_0$ such that $\alpha_0 S^\beta$ is a diffeomorphism φ_S^r of Γ_0. If $\Gamma_0 \times \Gamma_0$ is the coarse groupoid, S is the graph of φ^{-1}. The set $\hat{G}_\Gamma$ of bisections is a group and $S \to \varphi_S^r$ is an (anti) representation of $\hat{G}_\Gamma$ into Diffeo Γ_0, group of diffeomorphisms of Γ_0. $\hat{G}_\Gamma$ has another (anti) representation in Diffeo Γ, given by $S \to \phi_S^r$

$$\phi_S^r(x) = x.S = x.S^\beta(\alpha(x)).$$

We note by G_Γ the subgroup of $\hat{G}_\Gamma$ of bisections S such that φ_S^r has compact support, and we endowed G_Γ with a diffeological structure in Souriau's sense [15] that is we say that a map f from an open subset U of some $\mathbb{R}^p$ in G_Γ is "C^∞" if

(i) $U \times \Gamma_0 \to \Gamma$ $(u,x_0) \to \phi_{f(u)}^r(x_0) = f(u)(x_0)$ is C^∞.

(ii) For each $u_0 \in U$ there exists a compact K in Γ_0 and a neighbourhood V of u_0 in U such that for each $u \in V$, the support of $\varphi_{f(u)}^r$ is contained in K.

With this "diffeology", G_Γ is called the group of compactly controlled bisections.

If $E \to \Gamma_0$ is a Lie algebroid of anchor map ρ, we defined (mutatis mutandis) the Lie algebra $\mathfrak{K}_c(E)$ of compactly controlled sections s such that $\rho_0 s$ is a compactly supported vector field on Γ_0.

If G is a subgroup of G_Γ we can define the tangent space to G in Γ_0 as the set of time derivative for $t = 0$ of "C^∞" maps $f : I \to G$, where I is a neighbourhood of 0 in $\mathbb{R}$, such that $f(0) = \Gamma_0$. It turns that $\underline{G}_\Gamma = T_{\Gamma_0} G_\Gamma$ is $\mathfrak{K}_c(\underline{\Gamma})$ and that for any subgroup $G \subset G_\Gamma$, $\underline{G} = T_{\Gamma_0}G$ is a vector subspace of $\mathfrak{K}_c(\underline{\Gamma})$. We say that a vector subspace V of $\mathfrak{K}_c(\underline{\Gamma})$ is full if $T_0V = V$ where T_0V is the tangent space in 0. Moreover, for each $X \in \mathfrak{K}_c(\underline{\Gamma})$ we define $\exp X = \varphi_1^{X^\ell}(\Gamma_0)$ where $\varphi_t^{X^\ell}$ is the flow of the left invariant vector field X^ℓ associated to X. $t \to \exp tX$ is a C^∞ map from $\mathbb{R}$ into G_Γ .

Definition 3.1. *A subgroup* G *of* G_Γ *is an (infinite dimension) Lie group iff*

(i) $\underline{G} = T_{\Gamma_0}G$ *is full. Then* $\underline{G}$ *is a subLie algebra of* $\mathfrak{K}_c(\underline{\Gamma})$.

(ii) For each $X \in \underline{G}$ $t \to \exp tX$ *is a* C^∞ *map from* $\mathbb{R}$ *into* G.

Remark. If Γ_0 is reduced to a point G_Γ is a Lie group (of finite dimension) and Yamabe's theorem [17] implies that (i) and (ii) are always satisfied.

Let $(\Gamma,\mathcal{H})$ be a contact groupoid and $G^{\mathcal{H}}$ the set of *Legendrian* bisections.

Theorem 3.1. [4] $G^{\mathcal{H}}$ *is a Lie group.*

In order to describe $\underline{G}^{\mathcal{H}}$, we assume that $(\Gamma,\mathcal{H})$ is oriented and we consider a subordinate pfaffian groupoid (Γ,λ) and the Jacobi structure (Γ_0,E_0,Λ_0) on Γ_0 .

Then $j^1 : C^\infty(\Gamma_0,\mathbb{R}) \to \mathfrak{K}_c(\underline{\Gamma}) = \mathfrak{K}_c(J^1(\Gamma_0,\mathbb{R}))$ is an injective Lie algebra morphism and we note $C_c^\infty(\Gamma_0,\mathbb{R})$ the subLie algebra of $C^\infty(\Gamma_0,\mathbb{R})$ of u such that $j^1u \in \mathfrak{K}_c(\underline{\Gamma})$ with the diffeology induced that is to say $f : U \to C_c^\infty(\Gamma_0,\mathbb{R})$ is "C^∞" iff $(u,x_0) \to f_u(x_0)$ is C^∞ and j^1f_u compactly controlled.

Corollary. *If* $\mathcal{H}$ *is oriented,*

$$\underline{G}^{\mathcal{H}} = C_c^\infty(\Gamma_0,\mathbb{R}).$$

Remark. For an extension of these results if $\mathcal{H}$ is not oriented, cf. [4]. This extension solves the problem of integration of all Kirillov Lie algebras of rank one (cf. [8]).

4. PREQUANTIZATION

We restrict ourselves now to a special case of Jacobi manifolds, the Poisson ones in which $E_0 \equiv 0$. Then if (Γ_0,Λ_0) is contact integrable, the contact groupoid is always orientable and we can choose the contact form λ such that

$$\lambda(X_x \oplus Y_y) = \lambda(X_x) + \lambda(Y_y).$$

This peculiar situation has been studied by Libermann [11].

In this case the Reeb vector field E of (Γ,λ) is right and left invariant and so, E is complete. Then we can define a Lie groupoid morphism ψ in a trivial sense from the vector groupoid $\mathbb{R} \times \Gamma_0 \to \Gamma_0$ into Γ by mean of the E-flow, φ_t^E :

$$\begin{aligned} &\psi : \mathbb{R} \times \Gamma_0 \to \Gamma \\ &\quad \psi(t,x_0) = \varphi_t^E(x_0). \end{aligned}$$

ψ is an immersion and $\psi^*\lambda = dt$.

So if ψ is an embedding, λ restricted to $\mathfrak{J}^E(\Gamma_0) = \psi(\mathbb{R} \times \Gamma_0)$ is a regular foliation, which in turns implies that all the maps $t \to \varphi_t^E(x)$ have same period T independant of x (by convention $T = 0$ if $t \to \varphi_t^E(x)$ is injective).

Then $\tilde{\Gamma} = \Gamma/\mathfrak{J}^E(\Gamma_0)$ is a groupoid with a nice structure : it is a symplectic groupoid with the symplectic form σ induced by $d\lambda$. This means that, on $\tilde{\Gamma}$,

$$\sigma(X_x^1 \oplus Y_y^1, X_x^2 \oplus Y_y^2) = \sigma(X_x^1 , X_x^2) + \sigma(Y_y^1 ,Y_y^2)$$

and $\Gamma \to \tilde{\Gamma}$ is a prequantization in Weinstein's sense.

We shall say that a Poisson manifold (Γ_0,Λ_0) is symplectic-integrable if it is the units of a symplectic groupoid $(\tilde{\Gamma},\sigma)$. The Lie algebroid of $\tilde{\Gamma}$, $\underline{\tilde{\Gamma}}$ is the so called Lie algebroid of the Poisson manifold (Γ_0,Λ_0) which, as fiber bundle, is $T^*\Gamma_0 \to \Gamma_0$, and for which the anchor map is $\Lambda_0^\#$ and the bracket $\{\xi,\eta\} = [\xi,\eta]_P$ (cf. supra). Then $\mathfrak{A}_c(\underline{\tilde{\Gamma}}) = \Omega_c^1(\Gamma_0)$ the Lie algebra of 1-form ξ such that $\Lambda_0^\# \xi$ has a compact support. We note $Z\Omega_c^1(\Gamma_0)$ and $B\Omega_c^1(\Gamma_0)$ respectively the subLie algebras of closed and exact one-forms, and $H^1_c(\Gamma_o,\mathbb{R})$ the quotient $Z\Omega^1_c(\Gamma_o,\mathbb{R})/B\Omega^1_c(\Gamma_o,\mathbb{R})$. Then the exact sequence of diffeological Lie algebras

$$0 \to B\Omega^1_c(\Gamma_o) \to Z\Omega^1_c(\Gamma_o) \to H^1_c(\Gamma_o,\mathbb{R}) \to 0$$

can be integrate in the exact sequence of Lie groups (in the sense of definition 3.1)

$$0 \to G^{ex} \to G^\sigma_o \xrightarrow{C} H^1_c(\Gamma_o,\mathbb{R})/D \to 0$$

where G^σ_o is the connected component of the Lie group of Lagrangian bisection and where G^{ex} is a sub Lie group the group of "hamiltonian isotopies" which is defined in the following way :

$S \in G^{ex}$ iff it exists a C^∞ map $t \to S_t$ from a neighbourhood I of 0 in $\mathbb{R}$ to G^σ such that

(i) $S_o = \Gamma_o \qquad S_1 = S$

(ii) If $X_t \in X^\ell(\Gamma)$ is the time dependant vector field defined by

$$\frac{d}{dt}\phi^r_{S_t} = X_t \circ \phi^r_{S_t}$$

then $\iota_{X_t}\sigma = -\alpha^* dH_t$ where $t \to H_t$ is a C^∞ map from I to $C^\infty_c(\Gamma_o,\mathbb{R})$.

C is defined by mean of an extended notion of Calabi's invariant and D is a discrete subgroup (in diffeological sense) of $H^1_c(\Gamma_o,\mathbb{R})$.

Remark. The above result is an extension of the one given in [3]. In fact there is no necessity of the assumptions of [3] to have it. The results extend Banyaga's one [1].

Assume now that (Γ_o,Λ_o) is prequantizable so it is contact and symplectic-integrable. Then we can integrate also the exact sequence

$$0 \longrightarrow \mathbb{R} \longrightarrow C^\infty_c(\Gamma_o,\mathbb{R}) \xrightarrow{d} B\Omega^1_c(\Gamma_o) \longrightarrow 0$$

in the exact sequence of Lie groups

$$0 \longrightarrow \mathbb{R}/T\mathbb{Z} \longrightarrow G^{\mathcal{H}}_o \longrightarrow G^{ex} \longrightarrow 0$$

where T (eventually 0) is the common period of the Reeb vector field of (Γ,λ).

In fact, as usual, if $T = 0$ we can always by using a suitable $\mathbb{Z}$-quotient of (Γ,Λ) assume that $T = 1$. So in any case, the prequantized case leads to the exact sequence

$$0 \longrightarrow \mathbb{S}^1 \longrightarrow G_o^{\mathcal{H}} \longrightarrow G^{ex} \longrightarrow 0$$
$$(\mathbb{S}^1 \simeq \mathbb{R}/T\mathbb{Z} \quad T > 0)$$

and prequantization appears as a $\mathbb{S}^1$ central extension of the hamiltonian isotopies.

5. CONCLUDING REMARKS

5.1. There exists Poisson manifolds which are contact integrable but not symplectic-integrable. For example if we look at $\mathfrak{g}^*$ dual of a semi-simple compact Lie algebra, and at $S = S(\mathfrak{g}^*)$ the unit sphere (for the Killing form), $S(\mathfrak{g}^*)$ is a Poisson manifold which is symplectic integrable iff $\mathfrak{g} = so(3)$ but is - trivially - always contact integrable, with $\Gamma = (T^*G\,|_S,\lambda)$ where G is a Lie group of Lie algebra $\mathfrak{g}$ and λ is the Liouville form.

Another example is given by $\Gamma_o = \mathbb{S}^2 \times \mathbb{R}$ with $(S^1 \times \{t\}, \sigma_o f(t))$ as symplectic leaves, where σ_o is the standard form on $\mathbb{S}^2$ and f is a C^∞ function which is every where non zero. Then $\Gamma = \mathbb{S}^3 \times \mathbb{S}^3/\mathbb{S}^1 \times T^*\mathbb{R}$ with $\lambda = [p_1^*\lambda_o - p_2^*\lambda_o] + tdu$ when the bracket denotes the image in $\mathbb{S}^3 \times \mathbb{S}^3/\mathbb{S}^1$ (i.e. quotiented by the diagonal action) of $p_1^*\lambda_o - p_2^*\lambda_o$ where λ_o is the contact form on $\mathbb{S}^3$ and where Γ has the product groupoid structure of the $\mathbb{S}^1$-quotient structure of the coarse groupoid $\mathbb{S}^3 \times \mathbb{S}^3$, by the vector groupoid $T^*\mathbb{R}$. $(\Gamma_o,\sigma_o f(t))$ is symplectic integrable iff f(t) is constant either a submersion, in which case Γ_o is isomorphic to an open subPoisson manifold of $so(3)^*$.

If we look at $\Gamma_o = \mathbb{S}^2 \times \mathbb{S}^1$ with $\sigma_o f(t)$ as above, $(\Gamma_o,\sigma_o f(t))$ is symplectic and contact integrable iff f is constant.

5.2. If (Γ_o,Λ_o) is a symplectic manifold (Γ_o,σ_o), it is prequantizable in Weinstein's sense iff it exists $T > 0$ and a covering $\widetilde{\Gamma}_o$ of Γ_o such that $(\widetilde{\Gamma}_o, T^{-1}\widetilde{\sigma}_o)$ is prequantizable in Souriau's sense.

5.3. On any regular Poisson manifold there exists a star-product [12] but even on a symplectic manifold, the problem of the L^2-representation (in asymptotic sense) of the $*$-product cannot be solve if a cohomological condition is not satisfied [5] [7].

From these three remarks, it seems that a nice category of Poisson manifold to study problems of quantization is the category of Poisson manifolds, a covering of which is contact integrable.

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